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The Impact-Induced Triggering of Hot Spots in Energetic/Explosive Materials Part 3: An Elliptic Air-Filled Inclusion Within an Unbounded Isotropic Plane

by Michael Grinfeld

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14. ABSTRACT

In solid explosive materials, the holes filled with gas can portray explosion-triggering hot spots. We explore the form factor on the generation of the hot spots. As a result, we consider the Eshelby approach in the two-dimensional problem of determining the thermoelastic field of an elastic plane weakened by an elliptic hole and the field with liquid or gaseous media.

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1. Introduction

The classical theory of ignition in energetic substances is based on the appearance of hot spots. Hot spots are the localized areas with temperatures much higher than the average. These spots can be triggered by different physical effects and processes. One of them is based on the mechanism of adiabatic loading. The classical theory of this mechanism for the liquid/gas media is presented in the monographs in references *1–4*. The main element of this mechanism is that the easily compressible gaseous bubbles under given pressure become hotter than the less compressible condensed liquids surrounding them. In the first paper of this study (*5*), we demonstrated that the classical theory should be essentially revised by considering the effects of the surface tension. The revision mostly focuses on the influence of the dissolved bubbles of small sizes.

In the second paper of this series (*6*), we explored spherical holes in solid materials. These holes can serve as the local hot spots. They are completely different mechanisms than the local temperature concentrator. The gas inside the hole plays no role in this mechanism. The key idea associated with this mechanism is as follows. When the flammable liquid is subjected to external impact, the pressure is totally transferred from the liquid to the dissolved bubble. The situation changes completely when the liquid is substituted with solid. The solid then plays the "shielding" role and, at least partially, precludes direct transfer of the external impact. Thus, the local adiabatic stressing of the solid remains the only source of the local temperature rise.

One key difference between liquid and solid energetic materials was emphasized in Grinfeld and Bjerke (*6*). Namely, at equilibrium, the gaseous bubble in the liquid always has the spherical shape. At the same time, it can and does have arbitrary shapes in solid substances. Thus, the analysis of ref *6* is insufficient. Therefore, in this third part, we explore the role of the form factor of the hole. In order to make a more feasible and transparent study, we limit ourselves with the two-dimensional case. We also dwell on the explicit analysis of the elliptic holes.

Our analysis reveals one element of key importance—the "shielding effect" disappears for the ellipses of small eccentricity. Thus, intensive heating of gases becomes important again to fill these holes. We establish explicit relevant equations/formulas. Also, we present the exact nonlinear formulation of the problem. The nonlinear system can be solved numerically. For those numerical solutions, the established exact solutions can be used for verification purposes.

2. Statement of the Problem

Exact Nonlinear Statement of the Problem

In figure 1, an isotropic matrix contains an elliptic inclusion, which usually plays the role of stress concentrator.

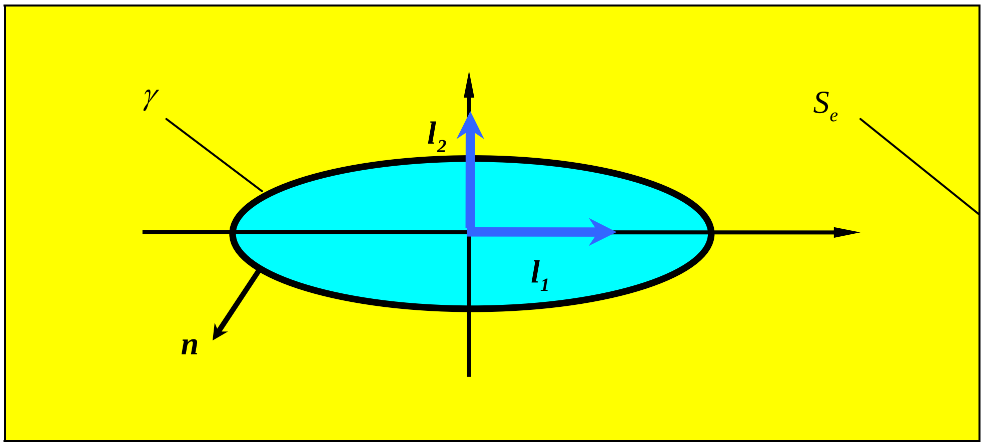

Figure 1. An elastic plane with a hole.

Our notation follows the monographs in references *7* and *8*. We shall use the notation $\vec{x}(x)$ and $\vec{X}(x)$ for the radius vectors of a material particle with Lagrangian coordinates x^i in the initial configuration and actual configurations, respectively. The radius vector $\vec{X}(x)$ can be presented in the following form:

$$\vec{X} = \vec{x} + \vec{u}(x), \tag{1}$$

where $\vec{u}(x)$ is the displacement vector.

Then, we use $\vec{x}_i(x) \equiv \partial\vec{x}(x)/\partial x^i$ and $\vec{X}_i(x) = \vec{x}_i + \partial\vec{u}/\partial x^i$ for the covariant bases in the initial and reference configurations. The notations $x_{ij} \equiv \vec{x}_i \cdot \vec{x}_j$ and $X_{ij} \equiv \vec{X}_i \cdot \vec{X}_j$ are used for the covariant metric tensors of the initial and reference configurations, respectively. For the contravariant metrics, we use x^{ij} and X^{ij}, respectively. We use $T^{i}_{.j\,|k}$ and $T^{i}_{.j\,||k}$ for the covariant differentiations of tensors in the initial and reference configurations. Tensors x_{ij}, x^{ij} are used for lowering and raising indexes.

We use $u^i(x)$, $u_i(x)$ for the components of displacements with respect to the basis in the initial configuration, i.e.,

$$\vec{u} = u^i \vec{x}_i \left(x \right) = u_i \vec{x}^i \left(x \right). \tag{2}$$

Note the following geometric relationships:

$$\vec{X}_j = a^{i\cdot}_{\cdot j} \vec{x}_i = \left(\delta^i_j + u^i_{\cdot|j} \right) \vec{x}_i \,,\; \vec{x}_j = b^{i\cdot}_{\cdot j} \vec{X}_j \,,\; a^{i\cdot}_{\cdot j} b^{j\cdot}_{\cdot k} \equiv \delta^i_k \,, \tag{3}$$

and

$$X_{ij} = x_{ij} + 2u_{ij} \,,\; u_{ij} = \frac{1}{2}\left(u_{i|j} + u_{j|i} + u^k_{\cdot|i} u_{k|j} \right), \tag{4}$$

where u_{ij} is the tensor of finite differences.

We use ρ_0 and ρ for the mass densities in the initial and actual configurations. In addition to their physical meaning, they make connections between the divergences of any contravariant tensor T^i in the initial and actual configurations:

$$T^i_{\cdot\|i} = \frac{\rho}{\rho_0} \left(\frac{\rho_0}{\rho} T^i \right)_{|i} . \tag{5}$$

We use P^{ij} and p^{ji} for the Cauchy-Green and Piola-Kirchhoff stress tensors, respectively. These two tensors are connected by the following relationship:

$$p^{ji} = \frac{\rho_0}{\rho} P^{jq} \left(\delta^i_q + u^i_{\cdot j|q} \right),\; P^{ji} = \frac{\rho}{\rho_0} p^{jq} \left(x_{qm} + u^{\cdot}_{q|m} \right) X^{mi} . \tag{6}$$

The exact universal bulk equilibrium equation within the matrix is

$$p^{ji}{}_{|i} = 0 . \tag{7}$$

The exact universal equilibrium equation at the interface matrix/gas is

$$\frac{\rho}{\rho_0} p^{jq} \left(x_{qm} + u^{\cdot}_{q|m} \right) X_{jk} X^{mi} n_i = -p_{gas} n_k . \tag{8}$$

When the matrix deforms elastically, the substance is fully characterized by the internal energy density $e\left(u_{i|j}, \eta\right)$ per unit volume in the initial configuration; η is the entropy density per unit volume in the initial configuration.

We assume that initial distribution of specific entropy η is uniform—$\eta(x) = \eta_{sol} = const$—and remains unchanged under the loading. We make the same assumption about the gas or liquid within the hole: $\eta(x) = \eta_{gas} = const$. In this case, exact equilibrium equations 7 and 8 can be rewritten in the following form:

$$\left(\frac{\partial e(u_{m|n}, \eta_{sol})}{\partial u_{i|j}} \right)_{|i} = 0 , \tag{9}$$

and

$$\frac{\rho}{\rho_0} \frac{\partial e(u_{m|n}, \eta_{sol})}{\partial u_{q|j}} \left(x_{qm} + u_{q|m} \right) X_{jk} X^{mi} n_i = -p_{gas} n_k . \tag{10}$$

We assume that displacements U^i are specified at the external boundary S_e of the heterogeneous solid:

$$u^i \Big|_{S_e} = U^i . \tag{11}$$

To make equations 9–11 mathematically closed, we also need to know the dependence of $p_{gas} = p_{gas}(V_{gas}, \eta_{gas})$.

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3. Semi-linear System of Adiabatic Loading

The system just presented is exact; however, it does not allow deeper mathematical analysis. It will be analyzed further by computer-based modeling. Computer modeling, in turn, requires comparison with some exact solutions and a qualitative understanding of the studied phenomenon. Therefore, we proceed with some simplifying assumptions. Namely, we concentrate on the phenomena for which the approximation of linear elasticity provides the physically adequate description of the matrix's behavior.

In the approximation of linear elasticity, equations 9 and 10 can be replaced with the following:

$${}_{\eta}C^{ijkl} u_{k,jl} = 0 , \tag{12}$$

and

$${}_{\eta}C^{ijkl} u_{k,l} n_j = -p\left(V_{gas}, \eta_{gas} \right) , \tag{13}$$

where ${}_{\eta}C^{ijkl}$ is the tensor of adiabatic modules.

Adiabatic loading is accompanied by the change $T_{ad} = \theta - \theta_0$ of the absolute temperature θ_0 , which is described by the following approximate relationship:

$${}_{ad}T = \frac{\theta_0}{C_u} B^{kl} u_{k,l} , \tag{14}$$

where C_u is the specific heat capacity at fixed deformations and B^{kl} is the tensor of coefficients of thermal expansion.

The actual volume of the gas-filled hole will be approximated by the following relation:

$$V_{gas} \approx V_0 - \int_S dS\, u^i n_i \,. \tag{15}$$

The case of isotropic matrix is characterized by the special structure of the tensors ${}_{\eta}C^{ijkl}$ and B^{kl}:

$$\begin{aligned} {}_{\eta}C^{ijkl} &= \lambda_\eta x^{ij} x^{kl} + \mu_\eta \left(x^{ik} x^{jl} + x^{il} x^{jk} \right) \\ &= \mu_\eta \left(\frac{2\nu_\eta}{1-2\nu_\eta} \delta^{ij}\delta^{kl} + \delta^{ik}\delta^{lj} + \delta^{il}\delta^{kj} \right), \ \nu_\eta \equiv \frac{\lambda_\eta}{2\left(\lambda_\eta + \mu_\eta\right)} \,; \\ B^{ij} &= -K_\theta \alpha x^{ij} \,, \end{aligned} \tag{16}$$

where λ_η and μ_η are the Lame adiabatic elastic modules (λ_θ and $\mu_\theta = \mu_\eta \equiv \mu$ are the Lame isothermal elastic modules), $K_\theta \equiv \lambda_\theta + 2\mu_\theta / 3$ is the isothermal module of volumetric compression, and α is the coefficient of thermal expansion.

For isotropic substances, we arrive at the following relationships:

$${}_{ad}T = -\frac{\theta_0}{C_u} K_\theta \alpha u^{k\cdot}_{\cdot k} \,, \tag{17}$$

and

$${}_{ad}\,p^{ij} = \lambda_\eta x^{ij} u^{k}_{\cdot,k} + 2\mu_\eta u^{i\ j}_{(\cdot,\cdot)} \,, \tag{18}$$

the second of which can also be presented in the following form:

$${}_{ad}\,p^{ij} = \left(\lambda_\theta + \frac{\alpha^2 K_\theta^2 \theta_0}{C_u} \right) u^{k\cdot}_{\cdot k} x^{ij} + 2\mu_\theta u^{i\ j}_{(\cdot,\cdot)} \,. \tag{19}$$

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4. An Elliptic Inclusion in a Stressed Isotropic Plane

In this section, we focus on one rich class of problems—permitting explicit solutions. These problems deal with an elliptic hole within an infinite isotropic matrix, with specified displacement gradients κ_{ij} at infinity.

$$u_i \to \kappa_{ij} x^j \ \ at \ \ | x^m | \to \infty \,. \tag{20}$$

We will be looking for solutions of the system (equation 20) next.

Within the matrix:

$$u_i = \kappa_{ij} x^j - \frac{1}{2\pi\mu} \tau^{j\cdot}_{\cdot i} \phi_{,j} + \frac{1}{8\pi\mu(1-\nu_\eta)} \tau^{jk}_{\cdot\cdot} \xi_{,ijk} , \tag{21}$$

Within the inclusion:

$$u_i = \omega_{ij} x^j , \tag{22}$$

where τ^{jk} is a certain symmetric matrix and $\phi(x)$ and $\xi(x)$ are the harmonic and bi-harmonic potentials of an ellipse having the shape of the hole:

$$\phi(x) = \int_\omega d\omega^* \ln\frac{1}{r},\ \zeta(x) = \int_\omega d\omega^* \frac{r^2}{2}\left(\ln\frac{1}{r}+1\right), r(x,x^*) \equiv |\vec{x}-\vec{x}^*| . \tag{23}$$

The potential $\phi(x)$ and its first derivatives are continuous across the ellipse boundary. The same is true about the potential $\xi(x)$ and its derivatives up to third order. The following relationships for discontinuities of the higher derivatives are valid:

$$\left[\phi_{,ij}\right]_-^+ = 2\pi n_i n_j ,\ \left[\xi_{,ijkl}\right]_-^+ = 4\pi n_i n_j n_k n_l . \tag{24}$$

Inside the ellipse, the second derivatives of the potential ϕ are constant and given by the following formula:

$$\phi_{,ij} = -N_{ij} = -\frac{2\pi a_1 a_2}{a_1 + a_2} \sum_{A=1}^{2} \frac{1}{a_A} l_{Ai} l_{Aj} , \tag{25}$$

where a_A is the length of the semi-axes of the ellipse and l_{Ai} is their director.

The analogous formula for the fourth derivatives of the potential $\xi(x)$ inside the ellipse is as follows:

$$\xi_{,ijkl} = -M_{ijkl} = -\frac{2\pi a_2 (a_1 + 2a_2)}{(a_1+a_2)^2} l_{1i} l_{1j} l_{1k} l_{1l} - \frac{2\pi a_1 a_2}{(a_1+a_2)^2} L_{ijkl} - \frac{2\pi a_1 (a_2 + 2a_1)}{(a_1+a_2)^2} l_{2i} l_{2j} l_{2k} l_{2l} , \tag{26}$$

where the tensor L_{ijkl} is equal to

$$L_{ijkl} = l_{1i} l_{1j} l_{2k} l_{2l} + l_{1i} l_{2j} l_{1k} l_{2l} + l_{1i} l_{2j} l_{2k} l_{1l} + l_{2i} l_{1j} l_{1k} l_{2l} + l_{2i} l_{1j} l_{2k} l_{1l} + l_{2i} l_{2j} l_{1k} l_{1l} . \tag{27}$$

Direct verification gives us

$$M^{\,j}_{.jkl} = \frac{2\pi a_2\left(a_1+2a_2\right)}{\left(a_1+a_2\right)^2} l_{1k}l_{1l} + \frac{2\pi a_1 a_2}{\left(a_1+a_2\right)^2}\left(l_{2k}l_{2l}+l_{1k}l_{1l}\right) + \frac{2\pi a_1\left(a_2+2a_1\right)}{\left(a_1+a_2\right)^2} l_{2k}l_{2l}$$

$$= \frac{4\pi a_2}{a_1+a_2} l_{1k}l_{1l} + \frac{4\pi a_2}{a_1+a_2} l_{2k}l_{2l} = 2N_{kl}. \tag{28}$$

Combining equations 21 and 24–26, we get the following formula of the displacement's gradients on the ellipse's boundary:

$$u_{i,j}\Big|_S = \left(\kappa_{ij} - \frac{1}{2\pi\mu}\tau^{k\cdot}_{.i}\,\phi_{,jk} + \frac{1}{8\pi\mu(1-\nu)}\tau^{kl}_{..}\,\xi_{,ijkl}\right)_{S+0}$$

$$= \kappa_{ij} + \frac{1}{2\pi\mu}\tau^{k\cdot}_{.i}\,N_{jk} - \frac{1}{8\pi\mu(1-\nu)}\tau^{kl}_{..}\,M_{ijkl} - \frac{1}{\mu}\tau^{k\cdot}_{.i}n_j n_k + \frac{1}{2\mu(1-\nu)}\tau^{kl}_{..}n_i n_j n_k n_l\,. \tag{29}$$

Using equation 29, we arrive at the following formula of the stresses in the matrix:

$$p^{mn}_s = p^{mn}_\infty + \left[\frac{\nu_\eta}{2(1-\nu_\eta)}x^{mn}\frac{1}{\pi}N_{kl} - \frac{1}{\pi}\frac{1}{4(1-\nu_\eta)}M^{\,mn}_{..kl} + \frac{1}{\pi}N^{(n}_{.k}\delta^{m)}_l\right]\tau^{kl}_{..} - \tau^{km}_{..}n^n n_k$$

$$+ \frac{1}{1-\nu_\eta}\left(n^m n^n - \nu_\eta\delta^{mn}\right)n_k n_l \tau^{kl}_{..} - \tau^{kn}_{..}n^m n_k\ , \tag{30}$$

where

$$p^{mn}_\infty \equiv \mu\left(\frac{2\nu_\eta}{1-2\nu_\eta}x^{ij}x^{mn} + x^{im}x^{nj} + x^{in}x^{jm}\right)\kappa_{ij}\,. \tag{31}$$

Contracting equation 30 with n_m, we get

$$p^{mn}_s n_n = \left(\sigma^{mn}_\infty + A^{mn..}_{..kl}\tau^{kl}_{..}\right)n_n\,, \tag{32}$$

where

$$A^{mn..}_{..kl} \equiv \frac{\nu_\eta}{2(1-\nu_\eta)}x^{mn}\frac{1}{\pi}N_{kl} - \frac{1}{\pi}\frac{1}{4(1-\nu_\eta)}M^{\,mn}_{..kl} + \frac{1}{\pi}N^{(n}_{.k}\delta^{m)}_l - \delta^n_k\delta^m_l\,. \tag{33}$$

In view of the boundary condition (equation 13) and the arbitrariness of the normal orientation that equation 32 implies,

$$A^{mn..}_{..kl}\tau^{kl} + p^{mn}_\infty = -p(V)\delta^{mn}\,. \tag{34}$$

Equation 34 should be treated as a linear equation with respect to the unknown tensor τ^{kl}.

Taking the skew-symmetric part of equation 34 and using equation 33, we get

$$\tau_{[kl]} = 0\,. \tag{35}$$

Thus, the tensor τ_{kl} is necessarily symmetric. For the symmetric part of tensor $\tau_{(kl)}$, equation 34 obviously implies the following:

$$A^{(mn)\cdot\cdot}_{\cdot\cdot(kl)}\tau^{(kl)} = -p^{mn}_{\infty} - p(V)\delta^{mn}\,. \tag{36}$$

For the actual volume V, we get

$$V = V_0\left[1+\kappa^{i}_{.i} - \frac{1}{2\pi\mu}\tau^{ji}_{..}\,\phi_{,ij} + \frac{1}{8\pi\mu(1-\nu_\eta)}\tau^{jk}_{..}\,\xi^{i..}_{,.ijk}\right]. \tag{37}$$

Relationship 37 is implied by the following chain:

$$\begin{aligned}
V &= V_0 + \int_S dSu_{i+}n^i = V_0 + \int_S dS\left\{\kappa_{ij}x^j - \frac{1}{2\pi\mu}\tau^{j.}_{.i}\,\phi_{,j} + \frac{1}{8\pi\mu(1-\nu_\eta)}\tau^{jk}_{..}\,\xi_{,ijk}\right\}_{S+0} n^i \\
&= V_0 + \int_S dS\left\{\kappa_{ij}x^j - \frac{1}{2\pi\mu}\tau^{j.}_{.i}\,\phi_{,j} + \frac{1}{8\pi\mu(1-\nu_\eta)}\tau^{jk}_{..}\,\xi_{,ijk}\right\}_{S-0} n^i \\
&= V_0 + \int_{V_0} dV\left\{\kappa^{i}_{.i} - \frac{1}{2\pi\mu}\tau^{ji}_{..}\,\phi_{,ij} + \frac{1}{8\pi\mu(1-\nu_\eta)}\tau^{jk}_{..}\,\xi^{i..}_{,.ijk}\right\} \\
&= V_0\left[1+\kappa^{i}_{.i} - \frac{1}{2\pi\mu}\tau^{ji}_{..}\,\phi_{,ij} + \frac{1}{8\pi\mu(1-\nu_\eta)}\tau^{jk}_{..}\,\xi^{i..}_{,.ijk}\right].
\end{aligned} \tag{38}$$

Using formulas 24, and 28, we can rewrite equation 37 as follows:

$$\frac{V}{V_0} = 1+\kappa^{i}_{.i} + \frac{1-2\nu_\eta}{4\left(1-\nu_\eta\right)\pi\mu}\tau^{ji}_{..}\,N_{ij}\ , \tag{39}$$

as it follows from the following chain:

$$\begin{aligned}
\frac{V}{V_0} &= 1+\kappa^{i}_{.i} - \frac{1}{2\pi\mu}\tau^{ji}_{..}\,\phi_{,ij} + \frac{1}{8\pi\mu(1-\nu_\eta)}\tau^{jk}_{..}\,\xi^{i..}_{,.ijk} \\
&= 1+\kappa^{i}_{.i} + \frac{1}{2\pi\mu}\tau^{ji}_{..}\,N_{ij} - \frac{1}{4\pi\mu(1-\nu_\eta)}\tau^{jk}_{..}\,N_{jk} = 1+\kappa^{i}_{.i} + \frac{1-2\nu_\eta}{4\left(1-\nu_\eta\right)\pi\mu}\tau^{ji}_{..}\,N_{ij}.
\end{aligned} \tag{40}$$

The following explicit formula of $A^{(mn)\cdot\cdot}_{\cdot\cdot(kl)}$ is derived in appendix A:

$$A^{(mn)\cdot\cdot}_{\cdot\cdot(kl)} \equiv \frac{1}{2(1-\nu_\eta)}\frac{4a_1a_2}{\left(a_1+a_2\right)^2} l_1^{(m}l_2^{n)}l_{1(k}l_{2l)}$$
$$-\frac{2(1-\nu_\eta)a_1a_1+\left(1-2\nu_\eta\right)a_1a_2}{2(1-\nu_\eta)\left(a_1+a_2\right)^2} l_{1k}l_{1l}l_1^m l_1^n + \frac{2\nu a_2a_2-\left(1-2\nu_\eta\right)a_1a_2}{2(1-\nu_\eta)\left(a_1+a_2\right)^2} l_{1k}l_{1l}l_2^m l_2^n$$
$$+\frac{2\nu_\eta a_1a_1-\left(1-2\nu_\eta\right)a_1a_2}{2(1-\nu_\eta)\left(a_1+a_2\right)^2} l_{2k}l_{2l}l_1^m l_1^n - \frac{a_1a_2\left(1-2\nu_\eta\right)+2a_2a_2\left(1-\nu_\eta\right)}{2(1-\nu_\eta)\left(a_1+a_2\right)^2} l_{2k}l_{2l}l_2^m l_2^n. \qquad (41)$$

Let $B^{kl\cdot\cdot}_{\cdot\cdot pq}$ be a tensor symmetric in the upper and lower indexes such that

$$A^{(mn)\cdot\cdot}_{\cdot\cdot(kl)} B^{kl\cdot\cdot}_{\cdot\cdot pq} \equiv \delta_p^{(m}\delta_q^{n)} \ . \qquad (42)$$

Tensor $B^{kl\cdot\cdot}_{\cdot\cdot pq}$ is calculated explicitly in appendix A and is given by equations A-3 and A-4.

Then, equation 36 implies

$$\tau^{(pq)} = -B^{pq\cdot\cdot}_{\cdot\cdot mn}\left[p_\infty^{mn} + p(V)\delta^{mn} \right]. \qquad (43)$$

Using equation 33, we can rewrite equation 30 as follows:

$$\sigma_s^{mn} = \sigma_\infty^{mn} + \left(A^{mn\cdot\cdot}_{\cdot\cdot kl} + \delta_k^n\delta_l^m \right)\tau^{kl}_{\cdot\cdot} - \tau^{km}_{\cdot\cdot} n^n n_k + \frac{1}{1-\nu_\eta}\left(n^m n^n - \nu_\eta\delta^{mn} \right) n_k n_l \tau^{kl}_{\cdot\cdot} - \tau^{kn}_{\cdot\cdot} n^m n_k \ . \qquad (44)$$

At last, using equation 36, we can transform equation 44 as

$$p_s^{mn} = p_\infty^{mn} + \left(A^{mn\cdot\cdot}_{\cdot\cdot kl} + \delta_k^n\delta_l^m \right)\tau^{kl}_{\cdot\cdot} - \tau^{km}_{\cdot\cdot} n^n n_k + \frac{1}{1-\nu_\eta}\left(n^m n^n - \nu_\eta\delta^{mn} \right) n_k n_l \tau^{kl}_{\cdot\cdot} - \tau^{kn}_{\cdot\cdot} n^m n_k$$
$$= -p(V)\delta^{mn} + \tau^{nm}_{\cdot\cdot} - 2\tau^{k(m}_{\cdot\cdot} n^{n)} n_k + \frac{1}{1-\nu_\eta}\left(n^m n^n - \nu_\eta\delta^{mn} \right) n_k n_l \tau^{kl}_{\cdot\cdot}. \qquad (45)$$

Let us consider any vector t_m tangential to the unit normal n_i. Then, equation 45 gives us

$$p_s^{mn} t_m t_n = -p(V) + \tau^{mn}_{\cdot\cdot}\left(t_m t_n - \frac{\nu_\eta}{1-\nu_\eta} n_m n_n \right). \qquad (46)$$

Using equations A-3 and A-4 from appendix A, we can rewrite equation 49 as follows:

$$\tau^{(kl)} = -B^{kl\cdot\cdot}_{\cdot\cdot mn}\left[p_\infty^{mn} + p(V)\delta^{mn} \right]$$
$$= -\begin{pmatrix} B_{11}l_1^k l_1^l l_{1p} l_{1q} + B_{12}l_1^k l_1^l l_{2p} l_{2q} + B_{21}l_2^k l_2^l l_{1p} l_{1q} + \\ B_{22}l_2^k l_2^l l_{2p} l_{2q} + b l_1^{(k} l_2^{l)} l_{1(p} l_{2q)} \end{pmatrix}\left[\sigma_\infty^{pq} + p(V)\delta^{pq} \right], \qquad (47)$$

or else as

$$\tau^{(kl)} = -B_{11} l_1^k l_1^l \left(\sigma_\infty^{pq} l_{1p} l_{1q} + p \right) - B_{12} l_1^k l_1^l \left(\sigma_\infty^{pq} l_{2p} l_{2q} + p \right)$$
$$- B_{21} l_2^k l_2^l \left(\sigma_\infty^{pq} l_{1p} l_{1q} + p \right) - B_{22} l_2^k l_2^l \left(\sigma_\infty^{pq} l_{2p} l_{2q} + p \right) - b l_1^{(k} l_2^{l)} l_{1(p} l_{2q)} \sigma_\infty^{pq}. \quad (48)$$

Equation 39 now takes the following form:

$$\frac{V - V_0}{V_0} = \frac{1}{\mu} \frac{a_1^3 + a_2^3}{a_1 a_2 (a_1 + a_2)} p\left(\frac{V}{V_0}\right) + p^i_{.i\infty} \frac{1 - 2\nu_\eta}{2\mu}$$
$$+ \frac{2(1-\nu_\eta) a_2^3 + (1 - 2\nu_\eta)(a_1 a_2^2 - a_1^2 a_2) + 2\nu a_1^3}{2\mu a_1 a_2 (a_1 + a_2)} p_\infty^{pq} l_{1p} l_{1q}$$
$$+ \frac{2(1-\nu_\eta) a_1^3 + (1 - 2\nu_\eta)(a_2 a_1^2 - a_2^2 a_1) + 2\nu a_2^3}{2\mu a_1 a_2 (a_1 + a_2)} p_\infty^{pq} l_{2p} l_{2q}. \quad (49)$$

Introducing the eccentricity $a_2 / a_1 \equiv \varepsilon$, we can rewrite equation 49 as follows:

$$\frac{V - V_0}{V_0} - \frac{1}{\mu} \frac{1 - \varepsilon + \varepsilon^2}{\varepsilon} p\left(\frac{V}{V_0}\right) = p^i_{.i\infty} \frac{1 - 2\nu_\eta}{2\mu} + \frac{(1-\nu_\eta)\varepsilon^3 + \nu_\eta}{\varepsilon(1+\varepsilon)} \frac{p_\infty^{pq}}{\mu} l_{1p} l_{1q}$$
$$+ \frac{1 - \nu_\eta + \nu_\eta \varepsilon^3}{\varepsilon(1+\varepsilon)} \frac{p_\infty^{pq}}{\mu} l_{2p} l_{2q} + \frac{(1 - 2\nu_\eta)(1-\varepsilon)}{1+\varepsilon} \frac{p_\infty^{pq}}{2\mu} \left(l_{2p} l_{2q} - l_{1p} l_{1q} \right). \quad (50)$$

---

5. Some Special Cases

In the case of a circular inclusion, eccentricity ε is equal to 1 and equation 50 reads

$$\frac{V - V_0}{V_0} - \frac{1}{\mu} p\left(\frac{V}{V_0}\right) = \frac{1 - \nu_\eta}{\mu} p_\infty^{ij} \left(l_{1i} l_{1j} + l_{2i} l_{2j} \right), \quad (51)$$

or else

$$\frac{V - V_0}{V_0} = \frac{1}{\mu} p\left(\frac{V}{V_0}\right) + \frac{1 - \nu_\eta}{\mu} \left(\sigma_{1\infty} + \sigma_{2\infty} \right), \quad (52)$$

where $\sigma_{1\infty}$ and $\sigma_{2\infty}$ are the principal stresses at infinity.

$$\sigma_{1\infty} \equiv p_\infty^{ij} l_{1i} l_{1j}, \; \sigma_{2\infty} \equiv p_\infty^{ij} l_{2i} l_{2j}. \quad (53)$$

At $a_2 / a_1 \equiv \varepsilon << 1$, we can reduce equation B-3 in appendix B to the following form:

$$\frac{V-V_0}{V_0}-\frac{1}{\varepsilon\mu}p\left(\frac{V}{V_0}\right)=\frac{\left(1-2\nu_\eta\right)\varepsilon+\nu_\eta}{\varepsilon}\frac{\sigma_{1\infty}}{2\mu}+\frac{\left(1-2\nu_\eta\right)\varepsilon+1-\nu_\eta}{\varepsilon}\frac{\sigma_{2\infty}}{2\mu}. \tag{54}$$

In the lowest order terms in ε, we get

$$p\left(\frac{V}{V_0}\right)=-\nu\sigma_{1\infty}-\left(1-\nu\right)\sigma_{2\infty}. \tag{55}$$

5.1 The "Linear" Gas Model

For further insight, let us consider the simplest case of a "linear gas" defined by the following relationship:

$$p=K_g\frac{V_0-V}{V_0}. \tag{56}$$

When dealing with the circular hole and the model of linear gas, equation 52 gives the following:

$$\frac{V_0-V}{V_0}=\frac{2\left(1-\nu_\eta\right)}{\mu+K_g}p_\infty, \tag{57}$$

and

$$p=\frac{2K_g\left(1-\nu_\eta\right)}{\mu+K_g}p_\infty, \tag{58}$$

where p_∞ is the pressure at infinity, defined as $p_\infty\equiv-\left(\sigma_{1\infty}+\sigma_{2\infty}\right)/2$.

Usually, the adiabatic incompressibility of gas K_g is much smaller than the shear modulus μ of the matrix. Then, equation 58 teaches us that there is a strong "shielding" factor Φ:

$$\Phi\equiv 2K_g\left(1-\nu_\eta\right)/\left(\mu+K_g\right). \tag{59}$$

In other words, the induced pressure p of the gas is much smaller than the pressure p_∞ acting at infinity, hence, the smaller induced temperature.

For the case of the arbitrary elliptic hole filled with the linear gas, equation 50 reads

$$p=\frac{\Gamma}{\Delta}\left(1-2\nu_\eta\right)\begin{bmatrix} p_\infty\varepsilon\left(1+\varepsilon\right)+\varepsilon\left(1-\varepsilon\right)\dfrac{\sigma_{1\infty}-\sigma_{2\infty}}{2}- \\ \sigma_{1\infty}\dfrac{\varepsilon^3-\nu_\eta\varepsilon^3+\nu_\eta}{1-2\nu_\eta}-\sigma_{2\infty}\dfrac{1-\nu_\eta+\nu_\eta\varepsilon^3}{1-2\nu_\eta}\end{bmatrix}, \tag{60}$$

where

$$\Gamma \equiv \frac{K_g}{\mu}, \Delta \equiv \frac{\left(1-\varepsilon+\varepsilon^2\right)\Gamma+\varepsilon}{1+\varepsilon}. \tag{61}$$

For finite Γ and $\varepsilon \to 0$, equations 60 and 61 give us the following in the lowest order terms:

$$p = -\nu_\eta \sigma_{1\infty} - \left(1-\nu_\eta\right)\sigma_{2\infty}. \tag{62}$$

Equation 62 shows that for the crack-like hole, the shielding effect of the matrix disappears completely; the pressure with the hole is the same order of magnitude as the stresses applied at infinity.

5.2 The Model of Ideal Gas

Equation 55 shows that the formula (equation 62) for the gas pressure is valid not only for the model of the "linear" gas within crack-like holes but, actually, for any gaseous liquid media.

We are reminded that the adiabatic processes of the ideal gases are described by the following relationships:

$$\frac{p}{p_0} = \left(\frac{V_0}{V}\right)^{\gamma}, \quad \frac{\theta}{\theta_0} = \left(\frac{V_0}{V}\right)^{\gamma-1}. \tag{63}$$

The constant γ is defined as $\gamma \equiv C_p / C_v$, where C_p and C_c are the specific heat capacities at fixed pressure and volume, respectively.

Thus, for this model, equation 50 can be rewritten as follows:

$$\left(\frac{\theta}{\theta_0}\right)^{\frac{1}{\gamma-1}} - \frac{p_0}{\mu}\frac{1-\varepsilon+\varepsilon^2}{\varepsilon}\left(\frac{\theta}{\theta_0}\right)^{\frac{\gamma}{\gamma-1}} = 1 - p_\infty \frac{1-2\nu_\eta}{\mu} + \frac{\left(1-\nu_\eta\right)\varepsilon^3+\nu_\eta}{\varepsilon\left(1+\varepsilon\right)}\frac{\sigma_{1\infty}}{\mu}$$
$$+ \frac{1-\nu_\eta+\nu_\eta\varepsilon^3}{\varepsilon\left(1+\varepsilon\right)}\frac{\sigma_{2\infty}}{\mu} + \frac{\left(1-2\nu_\eta\right)\left(1-\varepsilon\right)}{1+\varepsilon}\frac{\sigma_{2\infty}-\sigma_{1\infty}}{2\mu}, \tag{64}$$

where p_0 is the original pressure of gas in the hole.

6. Discussion and Conclusion

We formulated exact nonlinear systems (equations 9–11) to numerically explore the stress-induced hot spots around holes of different shapes filled with gases or liquids.

We then formulated semi-linear systems (equations 12–15) based on the assumption of small deformations in the solid matrix. The “pressure-volume” relationship for the gas or liquid remains nonlinear.

With the help of the semi-linear system, we analytically and explicitly explored the problem of thermoelastic stresses and deformation in an unbounded isotropic matrix weakened by the gas- or liquid-filled inhomogeneity.

The analysis showed two opposite effects. First, in the case of a circular inclusion, there was the “shielding” effect. In other words, due to the large shear resistance of the matrix, the pressure of the gas was considerably smaller than the stresses in the matrix. This effect does not exist when dealing with liquid energetic materials. Therefore, contrary to the case of liquid energetic substances, when dealing with solids, the spherical holes had a much less chance of becoming hot spots (at least, for relatively small stresses).

In the crack-like holes, the “shielding” effects from shear resistance disappeared. The gas pressure within the hole was the same order of magnitude as the stresses in the matrix. Therefore, the behavior of gases within these holes was completely analogous to the behavior of gases within spherical bubbles in liquids. The gas-filled holes in solid explosive materials had better chances of becoming the “hot” spots.

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Appendix A. A Closer Look at the Tensor $A^{(mn)\cdot\cdot}_{\cdot\cdot(kl)}$ and Its Symmetric Inverse $B^{kl\cdot\cdot}_{\cdot\cdot pq}$

Tensor $A^{(mn)\cdot\cdot}_{\cdot\cdot kl}$ can be presented in the following form:

$$A^{(mn)\cdot\cdot}_{\cdot\cdot(kl)} = A_{11} l_1^m l_1^n l_{1k} l_{1l} + A_{12} l_1^m l_1^n l_{2k} l_{2l} + A_{21} l_2^m l_2^n l_{1k} l_{1l} + A_{22} l_2^m l_2^n l_{2k} l_{2l} + a l_1^{(m} l_2^{n)} l_{1(k} l_{2l)} . \quad \text{(A-1)}$$

The set

$$G \equiv \left\{ l_1^m l_1^n l_{1k} l_{1l} \, , \, l_1^m l_1^n l_{2k} l_{2l} \, , \, l_2^m l_2^n l_{1k} l_{1l} \, , \, l_2^m l_2^n l_{2k} l_{2l} \, , \, l_1^{(m} l_2^{n)} l_{1(k} l_{2l)} \right\} \quad \text{(A-2)}$$

is called canonical basis.

Let $B^{kl\cdot\cdot}_{\cdot\cdot pq}$ be the inverse tensor with respect to $A^{(mn)\cdot\cdot}_{\cdot\cdot kl}$ in the sense of equation 42 from the body of this report. We assume that the tensor $B^{kl\cdot\cdot}_{\cdot\cdot pq}$ is symmetric with respect to the upper and lower indexes. We will be looking for $B^{kl\cdot\cdot}_{\cdot\cdot pq}$ in the same form as follows:

$$B^{kl\cdot\cdot}_{\cdot\cdot pq} = B_{11} l_1^k l_1^l l_{1p} l_{1q} + B_{12} l_1^k l_1^l l_{2p} l_{2q} + B_{21} l_2^k l_2^l l_{1p} l_{1q} + B_{22} l_2^k l_2^l l_{2p} l_{2q} + b l_1^{(k} l_2^{l)} l_{1(p} l_{2q)} . \quad \text{(A-3)}$$

Direct calculations lead to the following formula of the coefficients $B_{11}, B_{12}, B_{21}, B_{22}$, and b:

$$B_{11} = -\frac{1-\nu}{1-2\nu}\frac{2(1-\nu)a_2 + (1-2\nu)a_1}{a_1}, B_{12} = -\frac{1-\nu}{1-2\nu}\frac{2\nu a_2 - (1-2\nu)a_1}{a_1},$$
$$B_{21} = -\frac{1-\nu}{1-2\nu}\frac{2\nu a_1 - (1-2\nu)a_2}{a_2}, B_{22} = -\frac{1-\nu}{1-2\nu}\frac{a_2(1-2\nu) + 2a_1(1-\nu)}{a_2},$$
$$b = 2(1-\nu)\frac{(a_1+a_2)^2}{a_1 a_2}. \quad \text{(A-4)}$$

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Appendix B. Derivation of Equation 50

Combining equation 48 from the body of this report and equations A-3 and A-4 from appendix A, we get

$$
\begin{aligned}
\frac{1}{2\pi}\tau^{(kl)}N_{kl} &= -\frac{a_1 a_2}{a_1+a_2}\left(\frac{1}{a_1}l_{1k}l_{1l}+\frac{1}{a_2}l_{2k}l_{2l}\right) \\
&\quad\times\left\{\begin{array}{l} b l_1^{(k}l_2^{l)}l_{1(p}l_{2q)}p_\infty^{pq}+B_{11}l_1^k l_1^l\left(p_\infty^{pq}l_{1p}l_{1q}+p\right)+ \\ B_{12}l_1^k l_1^l\left(p_\infty^{pq}l_{2p}l_{2q}+p\right)+B_{21}l_2^k l_2^l\left(p_\infty^{pq}l_{1p}l_{1q}+p\right)+B_{22}l_2^k l_2^l\left(p_\infty^{pq}l_{2p}l_{2q}+p\right)+\end{array}\right\} \\
&= -\left(B_{11}\frac{a_2}{a_1+a_2}+B_{21}\frac{a_1}{a_1+a_2}\right)\left(\sigma_\infty^{pq}l_{1p}l_{1q}+p\right)-\left(B_{12}\frac{a_2}{a_1+a_2}+B_{22}\frac{a_1}{a_1+a_2}\right).
\end{aligned}
\tag{B-1}
$$

A straightforward calculation then gives us the following:

$$
\begin{aligned}
&\frac{1}{2\pi}\frac{1-2\nu_\eta}{1-\nu_\eta}\tau^{(kl)}N_{kl} \\
&= \frac{2\left(1-\nu_\eta\right)a_2^3+\left(1-2\nu_\eta\right)a_1a_2^2+2\nu_\eta a_1^3-\left(1-2\nu_\eta\right)a_1^2a_2+}{a_1a_2\left(a_1+a_2\right)}\left(p_\infty^{pq}l_{1p}l_{1q}+p\right) \\
&\quad+\frac{2\left(1-\nu_\eta\right)a_1^3+\left(1-2\nu_\eta\right)a_2a_1^2+2\nu_\eta a_2^3-\left(1-2\nu_\eta\right)a_2^2a_1+}{a_1a_2\left(a_1+a_2\right)}\left(p_\infty^{pq}l_{2p}l_{2q}+p\right).
\end{aligned}
\tag{B-2}
$$

Combining equation 39 from the body of this report with equation B-2, we arrive at the following equation:

$$
\begin{aligned}
\frac{V}{V_0} &= 1+\kappa_{.i}^{i}+\frac{1-2\nu_\eta}{4\left(1-\nu_\eta\right)\pi\mu}\tau_{..}^{ji}N_{ij} = 1+\kappa_{.i}^{i} \\
&\quad+\frac{2\left(1-\nu_\eta\right)a_2^3+\left(1-2\nu_\eta\right)a_1a_2^2+2\nu_\eta a_1^3-\left(1-2\nu_\eta\right)a_1^2a_2}{2\mu a_1a_2\left(a_1+a_2\right)}\left[p_\infty^{pq}l_{1p}l_{1q}+p\left(\frac{V}{V_0}\right)\right] \\
&\quad+\frac{2\left(1-\nu_\eta\right)a_1^3+\left(1-2\nu_\eta\right)a_2a_1^2+2\nu_\eta a_2^3-\left(1-2\nu_\eta\right)a_2^2a_1+}{2\mu a_1a_2\left(a_1+a_2\right)}\left[p_\infty^{pq}l_{2p}l_{2q}+p\left(\frac{V}{V_0}\right)\right].
\end{aligned}
\tag{B-3}
$$

Equation B-3 can be rewritten as follows:

$$\begin{aligned}\frac{V-V_0}{V_0} &= \kappa_{.i}^{i} \\ &+\frac{2\left(1-\nu_\eta\right)a_2^3+\left(1-2\nu_\eta\right)a_1a_2^2+2\nu_\eta a_1^3-\left(1-2\nu_\eta\right)a_1^2a_2}{2\mu a_1a_2\left(a_1+a_2\right)}\left[p_\infty^{pq}l_{1p}l_{1q}+p\left(\frac{V}{V_0}\right)\right] \\ &+\frac{2\left(1-\nu_\eta\right)a_1^3+\left(1-2\nu_\eta\right)a_2a_1^2+2\nu_\eta a_2^3-\left(1-2\nu_\eta\right)a_2^2a_1}{2\mu a_1a_2\left(a_1+a_2\right)}\left[p_\infty^{pq}l_{2p}l_{2q}+p\left(\frac{V}{V_0}\right)\right]. \qquad \text{(B-4)}\end{aligned}$$

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